# RS1, Custodial Isospin and Precision Tests

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**Abstract**

We study precision electroweak constraints within a RS1 model with gauge fields and fermions in the bulk. The electroweak gauge symmetry is enhanced to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, thereby providing a custodial isospin symmetry sufficient to suppress excessive contributions to the $T$ parameter. We then construct complete models, complying with all electroweak constraints, for solving the hierarchy problem, without supersymmetry or large hierarchies in the fundamental couplings. Using the AdS/CFT correspondence our models can be interpreted as dual to a strongly coupled conformal Higgs sector with *global* custodial isospin symmetry, gauge and fermionic matter being fundamental fields external to the CFT. This scenario has interesting collider signals, distinct from other RS models in the literature.

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# 1 Introduction

There is a puzzle at the heart of particle physics which has become ever sharper in the last two decades of experimental and theoretical research. The minimal Standard Model (SM) is thusfar in superb agreement with experiment, not just in terms of the central functions for which it was designed, but remarkably, in every accidental detail following from its minimality, such as suppressed flavor-changing neutral currents (FCNC's), proton stability, and a host of precision electroweak effects. Yet undeniably, the SM effective field theory suffers from the hierarchy problem and forces us to look beyond. Any approach for solving the hierarchy problem involves extending the SM at the weak scale and, in one way or another, threatens the economical and detailed agreement with experiment. Given this fundamental tension it is of considerable importance to identify within the different approaches to the hierarchy problem, robust effective field theory mechanisms which protect the key features of particle phenomenology, as well as the future experimental implications of these mechanisms.

The Randall-Sundrum I model (RS1) [1] [2] presented an exciting approach to the hierarchy problem based on geometrical hierarchies arising from warped higher-dimensional spacetime. However, most of the finer but interesting phenomenological issues are sensitive to the UV completion of the original RS1 effective field theory. The AdS/CFT correspondence [3] offers a great deal of insight into the RS1 proposal [4]. Via this correspondence, RS1 is dual to a purely 4D theory of particle physics and gravity, albeit one involving a strongly-coupled sector which is conformally invariant between the Planck and weak scales. The RS1 Kaluza-Klein excitations as well as fields localized on the "IR" brane are interpreted as TeV-scale composites of the strong sector. Fundamental fields coupled to strong CFT operators appear together as bulk RS fields. In the original RS1 model, all SM fields are localized on the IR brane, and therefore the model is dual to TeV-scale compositeness of the entire SM. The details of this compositeness determine the fate of the various phenomenological questions, but they are dual to details of the unknown UV completion of the RS1 effective theory.

There is another direction one can take. On the 4D side, to solve the hierarchy problem it is sufficient for just the Higgs to be a TeV-scale composite of a strongly interacting sector [5], the masses of higher-spin fundamental fields being protected by chiral or gauge symmetries. Of course for gauge boson and fermion masses to arise at the weak scale, the fundamental fields must couple to the Higgs sector. There is a simple way of studying this in the dual RS setting by continuing to take the Higgs to be localized on the IR brane, but taking gauge bosons and fermions to propagate in the higher dimensional bulk. The great advantage of doing this is that the key phenomenological issues become IR-dominated, and therefore addressable, in weakly-coupled RS effective field theory, rather than being sensitive to its UV completion. We

find this approach very exciting and promising.

Let us briefly review the history of such studies. With bulk gauge fields, the calculability of weak scale effects at first seemed a liability, with large harmful effects for compositeness [6] [7] [8] [9] [10] and precision electroweak observables [7] [8] [9] [10]. Reference [8] presented their results in terms of the Peskin-Takeuchi $S$ and $T$ parameters [11], which facilitated a global fit to the data. It was later realized that placing fermion fields in the bulk allowed one to greatly soften some of these effects [12] [7] [13] [14]. There were further dividends in that bulk fermion masses provided a simple attractive mechanism for generating Yukawa structure without fundamental hierarchies in the RS1 action [15] [12] [16]. Furthermore the same mechanism automatically protects the theory from excessive FCNC's [12] [16]. The issue of gauge coupling running and unification was discussed in Refs. [17] [18] [19] [20], with complete models with unification constructed in Ref. [20]. In particular a mechanism for protecting baryon stability was given in reference [20], adapting some key features of the mechanism of reference [21].

The last major phenomenological obstacle remaining in this program of research has been the problem of excessive contributions [7] [8] [9] to the Peskin-Takeuchi $T$ parameter [11]. The usual model-building rule for protecting this parameter is to ensure that the Higgs sector, when considered in isolation from gauge and fermion fields, should have a custodial isospin symmetry after electroweak symmetry breaking, under which the $W$'s form a triplet. However, the various RS1 models studied already appear to comply with this rule, since they make use of the minimal Higgs on the IR brane. However, the problem can be identified when one views these models from the dual 4D perspective. Bulk RS gauge fields are dual to both fundamental 4D gauge fields and to the CFT operators to which they might couple, namely conserved *global* symmetry currents of the CFT. This CFT represents the entire Higgs sector on the 4D side, of which the minimal Higgs is a light composite. The dual of the *entire* CFT Higgs sector enjoying a *global* custodial isospin symmetry is therefore to have a custodial isospin *gauge* symmetry in the RS *bulk*. Earlier RS models focused on *just* the SM gauge symmetry in the bulk. From the dual point of view, their difficulties with the $T$ parameter trace to the absence of custodial isospin symmetry in the CFT Higgs sector.

In this paper we study just such a bulk custodial isospin scenario and show that this extra gauge symmetry protects the $T$ parameter adequately. We are then able to construct fully realistic models satisfying all precision electroweak and other constraints. This is significant because we accomplish this in a non-supersymmetric approach to the hierarchy problem and without invoking any large fundamental hierarchies. In a composite Higgs model, the scale of compositeness can be made a free parameter. It can be raised at the cost of fine-tuning in the sense of the hierarchy problem, or lowered at the cost of strong interactions becoming more phenomenologically dangerous. The same is true in our RS model, where the scale of

Kaluza-Klein resonances is dual to the compositeness scale. Our fit to the body of precision test data requires such resonances to be above about 3 TeV.

An important consideration in this fit arises from the third generation quarks, especially the tension between the need to generate a large top quark mass while suppressing large corrections to bottom quark couplings to the $Z$. While some of the collider signals of our model are familiar expectations of strong interactions above the weak scale, some are more distinctively linked to the third generation constraints.

Our study illustrates the utility of RS effective field theory as a weakly coupled approach to a traditionally strongly-coupled and difficult subject: the possibility that the hierarchy problem is solved by non-supersymmetric physics above the weak scale. It allows us to calculate the signs and sizes of the leading effects on interesting observables in terms of model inputs, rather than just rough estimates. RS calculability is bought at a price. The dual strongly coupled theory must have special features [3] [4]: it must be approximately conformally invariant over the Planck-weak hierarchy, have a large-$N$ type expansion, and have a large gap in the spectrum of CFT scaling dimensions, with only a finite number being close to marginal (which automatically includes any symmetry CFT currents). Nevertheless, we have found that these special features do not pose any extra phenomenological liability, and are indeed an asset. Further, we expect many of our conclusions to survive even if some of the above theoretical control parameters are relaxed in Nature.

This paper is organized as follows. Section 2 describes the set-up of our model. Section 3 is a brief discussion of electroweak precision variables and the subtleties peculiar to our model. Sections 4 and 5 derive the tree level contributions to the Peskin-Takeuchi $S$ and $T$ parameters [11]. Section 6 derives the top loop contribution to $T$ in our model, bulk custodial isospin ensuring UV finiteness. Section 7 shows how our model can naturally fit the electroweak data. Section 8 describes the central new collider signals of our model. Section 9 describes the 4D dual CFT interpretation of our model and results. Section 10 provides further discussion and the outlook for future progress in this arena. Many of the more technical details have been relegated to appendices.

# 2 The Model

## 2.1 Overview

We are going to study a model with $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry in the bulk of a warped extra dimension. In order to recover the usual $SU(2)_L \times U(1)_Y$ we will break $SU(2)_R$ with orbifold boundary conditions on the Planck brane to $U(1)_R$, keeping the IR

brane $SU(2)_R$ symmetric. Then we will break $U(1)_R \times U(1)_{B_L} \to U(1)_Y$ spontaneously on the Planck brane. In one of the scenarios we consider, we will further break $SU(2)_R$ in the bulk by a small amount.

The metric of RS1 can be written as:

$$(ds)^2 = \frac{1}{(kz)^2}\big[\eta_{\mu\nu}dx^\mu dx^\nu - (dz)^2\big]. \tag{2.1}$$

Here,

$$\left(z_h \equiv \frac{1}{k}\right) \leq z \leq \left(z_v \equiv \frac{e^{k\pi r_c}}{k}\right), \tag{2.2}$$

where $k$ represents the $AdS_5$ curvature, $z_v \sim \mathrm{TeV}^{-1}$, and $\eta_{\mu\nu}$ is the 4D Minkowski metric. We take

$$k\pi r_c \sim \log\left(M_{Pl}/TeV\right) \sim 30 \tag{2.3}$$

to solve the hierarchy problem.

In that background the lagrangian for our model reads:

$$S = \int d^4x dz \sqrt{G}(\mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{\mathrm{UV}}\delta(z - z_h) + \mathcal{L}_{\mathrm{IR}}\delta(z - z_v)). \tag{2.4}$$

$\mathcal{L}_{gauge} + \mathcal{L}_{fermion}$ is the bulk lagrangian. We focus on $\mathcal{L}_{gauge}$ first, discussing $\mathcal{L}_{fermion}$ in section 2.4.

$$\begin{aligned}\mathcal{L}_{gauge} &= \sqrt{G}\big( - \frac{1}{4}TrW_{MN}W^{MN} - \frac{1}{4}Tr\widetilde{W}_{MN}\widetilde{W}^{MN} \\ &\quad -\frac{1}{4}Tr\widetilde{B}_{MN}\widetilde{B}^{MN} - \frac{1}{4}TrF_{MN}F^{MN} + |D_M\Sigma|^2 - V(\Sigma)\big),\end{aligned} \tag{2.5}$$

where the indices are contracted with the bulk metric $G_{MN}$, and $W^{MN}$ is field strength for the $SU(2)_L$ gauge group, $\widetilde{W}_{MN}$ for $SU(2)_R$, $\widetilde{B}_{MN}$ for $U(1)_{B-L}$ and $F_{MN}$ is for the gluon. $\Sigma$ is a triplet of $SU(2)_R$ whose sole purpose is to spontaneously break $SU(2)_R$ to $U(1)_R$ at a mass scale below $k$. Therefore, henceforth, we will simply work with the gauge theory with a mass term for $\tilde{W}^{\pm}$:

$$\begin{aligned}\mathcal{L}_{gauge} &= \sqrt{G}\big( - \frac{1}{4}TrW_{MN}W^{MN} - \frac{1}{4}Tr\widetilde{W}_{MN}\widetilde{W}^{MN} \\ &\quad -\frac{1}{4}Tr\widetilde{B}_{MN}\widetilde{B}^{MN} - \frac{1}{4}TrF_{MN}F^{MN} + \tilde{M}^2|\tilde{W}^{\pm}|^2\big)\end{aligned} \tag{2.6}$$

In fact, it will be interesting to consider 2 separate cases, *Scenario I*, where $\tilde{M}/k$ is small, but non-zero and *Scenario II*, where $\tilde{M}/k = 0$, i.e., $SU(2)_R$ is unbroken in the bulk.

$\mathcal{L}_{\mathrm{UV}}$ includes the necessary fields to spontaneously break $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y$ and $\mathcal{L}_{\mathrm{IR}}$ contains the SM Higgs field, now a *bi*doublet of $SU(2)_L \times SU(2)_R$:

$$\mathcal{L}_{\mathrm{IR}} = \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}, \tag{2.7}$$

where $\mathcal{L}_{Yukawa}$ will generate Yukawa couplings for fermions which will be discussed in section 2.4 and

$$\mathcal{L}_{Higgs} = \sqrt{-g_{\rm IR}}\left(D_\mu H\left[D^\mu H\right]^\dagger - V(H)\right). \tag{2.8}$$

$g_{IR}$ is the *induced* flat space metric in the IR brane. After the usual field redefinition of $H$ [1], Eq. (2.8) takes its canonical form:

$$\mathcal{L}_{Higgs} = D_\mu H\left[D^\mu H\right]^\dagger - V(H) \tag{2.9}$$

with $\langle H\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2}\end{pmatrix}$, $v \approx 250\text{GeV}$, and the ratio of the Higgs vev to the warped down curvature scale is taken as

$$v z_v \sim \frac{1}{5}. \tag{2.10}$$

We assume that brane-localized (kinetic) terms for bulk fields are of order loop processes involving bulk couplings and are therefore neglected in our analysis (however, see references [22, 23, 14] for effects of larger brane-localized kinetic terms for gauge fields and reference [24] for effects of brane-localized kinetic terms for fermions.).

### 2.2 $SU(2)_R \to U(1)_R$ by orbifold boundary condition

In addition to bulk breaking of $SU(2)_R$, we further break $SU(2)_R$ to $U(1)_R$ by orbifold boundary condition. Therefore, we assign the following boundary conditions to the $\mu$-components of the gauge fields under an $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold [25, 26, 27].

| | UV | IR |
|---|---|---|
| $\widetilde{W}_\mu^{1,2}$ | $-$ | $+$ |
| other gauge fields | $+$ | $+$ |

### 2.3 $U(1)_R \times U(1)_{B-L} \to U(1)_Y$ on UV brane

The breaking of $U(1)_R \times U(1)_{B-L} \to U(1)_Y$ occurs via a vev on the UV brane. There are two linear combinations of $\widetilde{W}_\mu^3$ and $\widetilde{B}_\mu$,

$$Z'_\mu \equiv \frac{\tilde{g}_5 \widetilde{W}_\mu^3 - \tilde{g}'_5 \widetilde{B}_\mu}{\sqrt{\tilde{g}_5^2 + \tilde{g}'^2_5}}, \qquad \text{and} \qquad B_\mu \equiv \frac{\tilde{g}'_5 \widetilde{W}_\mu^3 + \tilde{g}_5 \widetilde{B}_\mu}{\sqrt{\tilde{g}_5^2 + \tilde{g}'^2_5}}, \tag{2.11}$$

where

$$D_M = \partial_M - i(g_5 W_M^a \tau_{a\,L} + \tilde{g}_5 \widetilde{W}_M^a \tau_{a\,R} + \tilde{g}'_5 \widetilde{B}_M \widetilde{Y}), \tag{2.12}$$

is the electroweak covariant derivative with $\widetilde{Y} = \frac{B-L}{2}$. $B_\mu$ is the hypercharge gauge boson. It is $(+,+)$. We couple $Z'_\mu$ to a Planckian vev on the UV brane which mimics $(-,+)$ boundary condition to a good approximation.

In terms of $Z'$ and $B$, the five dimensional electroweak covariant derivative is now

$$D_M \;\; = \;\; \partial_M - i\big(g_5 W^a_M \tau_{a\,L} + \tilde{g}_5 \widetilde{W}^{1,2}_M \tau^{1,2}_R + g_{5Z'} Z'_M Q_{Z'} + g'_5 B_M(\tau^3_R + \widetilde{Y})\big). \tag{2.13}$$

We have defined the hypercharge coupling,

$$g'_5 = \frac{\tilde{g}'_5\, \tilde{g}_5}{\sqrt{\tilde{g}_5^2 + \tilde{g}_5'^{\,2}}}, \tag{2.14}$$

the $Z'$ charge

$$Q_{Z'} = \tau^3_R - \sin^2\theta'\, Y, \tag{2.15}$$

the $Z'$ coupling

$$g_{Z'\,5} = \sqrt{\tilde{g}_5^2 + \tilde{g}_5'^{\,2}}, \tag{2.16}$$

and the $\widetilde{B} - \widetilde{W}^3$ mixing angle

$$\sin\theta' = \frac{\tilde{g}'_5}{g_{Z'\,5}}. \tag{2.17}$$

## 2.4 Fermions

Since we have an enhanced bulk gauge symmetry, namely $SU(2)_R$, we have to promote the usual right handed fermionic fields to doublets of this symmetry. Moreover, since we are breaking that symmetry through the UV orbifold, one component of $SU(2)_R$ doublet must be even and therefore has a zero-mode while the other component must be odd and therefore does not have a zero-mode. Therefore, we are forced to double the number of right handed doublets in such a way that from one of them the upper component, for example up type quark, is even whereas from the other the lower component, down type, is even – this is similar to obtaining quarks and lepton zero-modes from different $SU(5)$ bulk multiplets in orbifolded GUT scenarios [26, 27]. This doubling of right handed particles is only needed in the quark sector, since in the lepton sector we only need the right handed charge leptons[1] to be massless after we compactify. Explicitly, we have three types of doublets under $SU(2)_R$ per generation in such a way that[2]:

$$\begin{aligned} Q_{R\,1} &= u_R + d'_R \\ Q_{R\,2} &= u'_R + d_R \\ L_R &= e_R + \nu'_R \end{aligned} \tag{2.18}$$

[1]Although in any embedding of this theory in a GUT, the minimal group would be $SO(10)$, thus needing a $\nu_R$ and another doublet.

[2]Only one chirality will be discussed since the other chirality is projected out by $\mathbb{Z}_2$ symmetry.

where the *un*primed particles are the ones to have zero modes, i.e. to be $(+,+)$. The extra fields needed to complete all representations are $(-,+)$ (since breaking of $SU(2)_R$ is on the Planck brane).

The general bulk lagrangian for fermions is:

$$\mathcal{L}_{fermion} = \sqrt{G}(i\bar{\Psi}\Gamma^M D_M \Psi - \epsilon(y) c_\Psi \bar{\Psi}\Psi) \tag{2.19}$$

where $\epsilon(y)$ is the sign function. Even though it will seem that we are adding a mass term, $c_\Psi$ is compatible with a massless zero mode. This parameter controls the localization of the zero mode, for $c > 1/2$ ($c < 1/2$) the wavefunction near the Planck (IR brane) [15, 12].

The Yukawa couplings to Higgs (prior to field redefinition of Eq. (2.8) $\rightarrow$ Eq. (2.9)) are necessarily localized on the IR brane:

$$\mathcal{L}_{Yukawa} = \sqrt{-g_{IR}} H \left( \lambda_{u\,5} Q_L Q_{R\,1} + \lambda_{d\,5} Q_L Q_{R\,2} + \lambda_{e\,5} L_L L_R \right) \tag{2.20}$$

Note that because $u_R$ and $d_R$ zero-modes arise from *different* $SU(2)_R$ doublets, we are able to give them separate Yukawa couplings without violating $SU(2)_R$ on the IR brane.

So far, we have detailed the model, except for choice of $c$'s. The $c$ parameters give a simple mechanism for obtaining hierarchical $4D$ Yukawa couplings with*out* hierarchies in $5D$ Yukawa couplings. In short, light fermions are localized near *Planck* brane ($c > 1/2$) so that their $4D$ Yukawa couplings are small due to the small overlap with Higgs on *TeV* brane. Left-handed top and bottom quarks are close to $c = 1/2$ (but $< 1/2$)[3], whereas *right*-handed top quark is localized near TeV brane to get $O(1)$ top Yukawa. With this set-up, FCNC's from exchange of both gauge KK modes and "string states" (parameterized by flavor-violating local operators in our effective field theory) are also suppressed. See references [12, 16] for details.

# 3 Electroweak precision observables

We begin with formalism for electroweak fit in the presence of new physics. It is convenient to discuss this in the framework of $4D$ effective Lagrangian at the weak scale for SM with all the heavy non-standard physics integrated out [28] (as pioneered in references [29], but here retaining the Higgs field). This framework was used earlier in the RS model studied in reference [8]. The dimension-6 operators, obtained by integrating out heavy particles, which are important for the electroweak fit are:

$$\mathcal{L}_{gauge-kinetic} = \frac{g g' s}{16\pi^2 v^2} H^\dagger \tau^a H B^{\mu\nu} W_{a\,\mu\nu} \tag{3.1}$$

[3] As we will show $c_L \sim 1/2$ is necessary to be consistent with $Z \rightarrow \bar{b}_L b_L$ for KK masses $\sim$ few TeV.

$$\mathcal{L}_{gauge-mass} = \frac{-t}{16\pi^2 v^2}\left[ (D^\mu H)^\dagger H\right] \left(H^\dagger D_\mu H\right) \tag{3.2}$$

$$\mathcal{L}_{fermion} = \frac{-ix}{16\pi^2 v^2}\bar{\psi}\gamma^\mu \tau^a \psi \left(D_\mu H\right)^\dagger \tau_a H + \frac{-iy}{16\pi^2 v^2}\bar{\psi}\gamma^\mu \psi \left(D_\mu H\right)^\dagger H + \frac{V}{16\pi^2 v^2}\bar{\psi}\psi\bar{\psi}\psi + h.c., \tag{3.3}$$

where $x$, $y$ and $V$, in general, vary with the fermion.

Usually, the gauge-kinetic higher-dimensional operator in Eq. (3.1) and the (custodial-isospin violating) mass higher-dimensional operator in Eq. (3.2) translate into "oblique" parameters, $S$ and $T$ [11], respectively:

$$\begin{aligned} S &= \frac{s}{2\pi} \\ T &= \frac{t}{8\pi e^2}, \end{aligned} \tag{3.4}$$

while the fermionic operators in Eq. (3.3) are considered "non-oblique". However, for a special form of fermion-*Higgs* higher-dimensional operators in Eq. (3.3), these can be field-redefined into *purely* oblique effects as we now discuss. In the particular RS models studied in references [8,23], with all fermions on the IR brane, the equivalent of our field redefinition was achieved by setting the gauge boson wavefunction to be unity on the IR brane. Reference [14] studied *bulk* fermions and used analogous field redefinitions.

This special form is

$$\begin{aligned} x &= ag^2 \\ y &= ag'^{\,2} Y Y^H \end{aligned} \tag{3.5}$$

for *all* fermions, where $Y = Q_{em} - \tau^3_L$ denotes the hypercharge of the fermion and $Y_H$ is the hypercharge of the Higgs. Setting Higgs to its vev in the fermion-Higgs operator in Eq. (3.3) induces non-canonical couplings of fermions to gauge bosons. However, doing the following redefinition of gauge fields renders the fermionic couplings to gauge bosons canonical[4]:

$$\begin{aligned} W_3 &\rightarrow W_3\left(1 - g^2\frac{a}{64\pi^2}\right) + Bgg'\frac{a}{64\pi^2} \\ W^\pm &\rightarrow W^\pm\left(1 - g^2\frac{a}{64\pi^2}\right) \\ B &\rightarrow B\left(1 - g'^{\,2}\frac{a}{64\pi^2}\right) + W_3 gg'\frac{a}{64\pi^2}. \end{aligned} \tag{3.6}$$

[4]This can be extended (non-linearly) into a manifestly $SU(2)_L \times U(1)_Y$-invariant redefinition.

This redefinition when substituted in SM gauge kinetic terms induces shifts in $s$ and $t$ i.e., purely oblique effects, so that we now have:

$$\begin{aligned} S &= \frac{s}{2\pi} + \frac{a}{2\pi} \\ T &= \frac{t}{8\pi e^2} + \frac{a g'^{\,2}}{8\pi e^2} \end{aligned} \tag{3.7}$$

As we will show, in our model, the fermion-Higgs operators have the special form only for the *light* fermions (and *right*-handed bottom), *not* for the top and *left*-handed bottom quark so that as far as the precision electroweak fit is concerned, couplings of $b_L$ have to be considered separately. Hence, we will focus on $S$ and $T$ parameters and $Z \to \bar{b}_L b_L$ in this paper.

In the following, we will integrate out Kaluza-Klein (KK) (*heavy*) modes of gauge/fermion fields in the RS1 model and compute the resulting dimension-6 operators for the (light) *zero*-modes (which correspond to the SM fields in the above Lagrangian). There are even higher-dimensional operators whose effect on electroweak precision observables can be considered. Naively, these are further suppressed by $\sim g^2 v^2 / m_{KK}^2$, where $m_{KK}$ is the KK mass scale. In the present scenario, these are also $k\pi r_c$-enhanced. Therefore, we are careful in what follows to consider KK mass scales such that even with this enhancement, these operators are suppressed relative to the dimension-6 operators. This justifies using a Higgs vev insertion approximation within gauge and fermions propagators in what follows.

# 4 Tree-level $S$ and $T$ contributions from gauge-Higgs sector

## 4.1 Contribution to $T$

A powerful aspect of our model is that the bulk right-handed gauge symmetry enforces custodial isospin. We will see that the gauge sector does not make a logarithmically enhanced contribution to the $T$ parameter, and that the $S$ parameter is log enhanced, but suppressed by $(v z_v)^2$ relative to $T$.

In the effective theory (section 3), the operator $t |H^\dagger D_\mu H|^2/(4\pi v)^2$ contributes only to the $W^3$ mass at order $v^4$. Consequently, the coefficient $t$ measures the amount of isospin breaking. In terms of vacuum polarizations, the effective theory contains a modified quadratic term,

$$\mathcal{L}_{\text{eff}} \supset g^2 \Big( \frac{v^2}{8} + \frac{\Pi_{aa}(0)}{2} \Big) W_\mu^{a\,(0)} W^{a\,(0)\mu}. \tag{4.1}$$

$\Pi_{ab}(q)$ is polarization from integrating out tree level insertions of gauge KK modes. Thus, from

Eqs. (3.2) and (4.1), we see that the coefficient of the operator $|H^\dagger D_\mu H|^2/(4\pi v)^2$ is

$$t = -\frac{128\pi^2}{v^2} \left(\Pi_{33}(0) - \Pi_{11}(0)\right). \tag{4.2}$$

In the gauge sector, oblique corrections to the electroweak observables come from integrating out KK towers which couple to left handed zero modes (Fig. 1). We will find it convenient to convert sums over KK propagators (eigenfunctions) to five dimensional propagators (Green's functions) (see appendix A), while leaving four dimensional fields on external lines (see appendix B for details).

We use Eq. (B.4) to calculate the contribution to $\Pi_{aa}(q)$ from Fig1,

$$ig^2\Pi_{aa}(q)\eta^{\mu\nu} = \sum_j i\frac{v^4}{16}g^2 g_{5i}^2 \left(G^{5D}_{q\,j}(z_v,z_v) - G^{(0)}_{q\,j}\right)\eta^{\mu\nu}, \tag{4.3}$$

where $g = g_5/\sqrt{\pi r_c}$ is the $4D$ or zero-mode gauge coupling. The sum over $j$ includes all fields which couple to the external $W^{a\,(0)}$. $G^{5D}_{qj}$ in (4.3) is the IR brane to IR brane, five dimensional, mixed momentum-position space Green's function in Feynman gauge. We *subtract* the massless pole, $G^{(0)}_{q\,j}(z_v,z_v)$, since the effective Lagrangian is obtained by integrating out *only* heavy/KK modes at tree-level (graphs with internal zero-modes are part of the Dyson resummation).

In the charged sector, the $W^{1\,(0)}$ mixes with its own KK modes and those of the $\widetilde{W}^{1\,(n)}$:

$$\Pi_{11}(q) = \left(\frac{v^2}{4}\right)^2 \left\{g_5^2\left(G^{5D}_{q(++)}(z_v,z_v) - G^{(0)}_{q(++)}\right) + \tilde{g}_5^2 G^{5D}_{q(-+)}(z_v,z_v)\right\}. \tag{4.4}$$

In the neutral sector, $W^{3\,(0)}$ mixes with its own KK modes, the $B$, and those of the $Z'$:

$$\Pi_{33}(q) = \left(\frac{v^2}{4}\right)^2 \left\{(g_5^2 + g_5'^2)\left(G^{5D}_{q(++)}(z_v,z_v) - G^{(0)}_{q(++)}\right) + g^2_{Z'\,5}\cos^4\theta' G^{5D}_{q(-+)}(z_v,z_v)\right\}. \tag{4.5}$$

The $\Pi$'s contain IR brane to IR brane propagators and we have used the $Z'$ charge of Higgs (see Eq. (2.15)).

Using the propagators in the appendix B, we find

$$\Pi_{11}(0) \approx \frac{-(k\pi r_c)g^2(vz_v)^2}{8}\left\{1 - \frac{1}{k\pi r_c} + \mathcal{O}\left(\frac{1}{(k\pi r_c)^2}\right)\right\}\frac{v^2}{4} - \frac{(k\pi r_c)\tilde{g}^2(vz_v)^2}{8}\left(1 - \frac{\tilde{M}^2}{4k^2}\right)\frac{v^2}{4}, \tag{4.6}$$

and

$$\Pi_{33}(0) \approx \frac{-(k\pi r_c)(g^2+g'^2)(vz_v)^2}{8}\left\{1 - \frac{1}{k\pi r_c} + \mathcal{O}\left(\frac{1}{(k\pi r_c)^2}\right)\right\}\frac{v^2}{4} - \frac{(k\pi r_c)(g^2_{Z'}\cos^4\theta')(vz_v)^2}{8}\frac{v^2}{4} \tag{4.7}$$

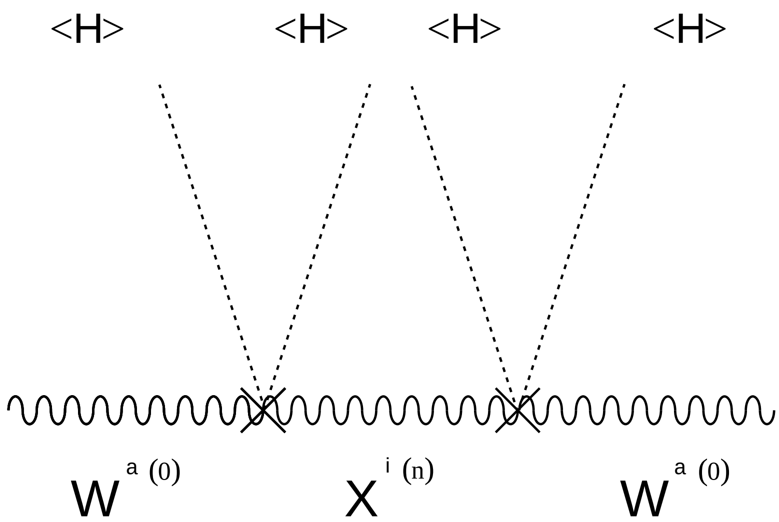


Figure 1: Contributions to $S$ and $T$ from gauge-Higgs sector at tree-level. As in appendix B, $X^i$ is any field coupling to the $W^a$'s.

where $g' = g'_5/\sqrt{\pi r_c}$, $\tilde{g} = \tilde{g}_5/\sqrt{\pi r_c}$ $g_{Z'} = g_{Z'\,5}/\sqrt{\pi r_c}$. In (4.6) and (4.7), we have also dropped terms at order $v^4$ that are $e^{-k\pi r_c}$ suppressed relative to the leading terms above. Then,

$$\Pi_{33}(0) - \Pi_{11}(0) \approx \frac{(g'^2)(vz_v)^2}{8}\frac{v^2}{4} - \frac{\tilde{M}^2}{4k^2}\frac{k\pi r_c}{8}\tilde{g}^2\,(vz_v)^2\,\frac{v^2}{4}, \quad \text{and} \tag{4.8}$$

$$t = 16\pi^2 v^2 z_v^2 \left(\frac{\tilde{M}^2}{4k^2}\frac{\tilde{g}^2}{4}k\pi r_c - \frac{g'^{\,2}}{4}\right). \tag{4.9}$$

Hence, in scenario II, the gauge sector *does not* contribute a log enhanced piece to custodial $SU(2)$ breaking at this order as would be the case in the absence of $SU(2)_R$.

## 4.2 Contribution to S

At tree-level, there are no Feynman diagrams that contribute to $s$ as defined in Eq. (3.1). As discussed at the end of section 3, there may be even higher-dimensional operators which can contribute to precision variables which we argued on general grounds are small in our model. However, in the case of the $S$ parameter since the dimension-6 contribution, $s$, vanishes, to be cautious, we will calculate dimension-8 contribution of the type depicted in Fig.1.

Since this is a tree-level calculation, there is no (kinetic) mixing involving the photon (of course, there can*not* be any mass mixing even at loop level), so the quantity

$$\left(\Pi'_{33}(q)|_{q^2=0} - \Pi'_{3Q}(q)|_{q^2=0}\right) \approx \Pi'_{33}(q)|_{q^2=0}, \tag{4.10}$$

is related to $S$, with

$$\Pi'_{33}(q)|_{q^2=0} \approx \frac{-(g^2+g'^2+4g^2_{Z'H})(k\pi r_c)(vz_v)^4}{256} + \mathcal{O}(v^4 z_v^4). \tag{4.11}$$

Eq. (4.11) is equivalent to a shift in $S$ parameter, $\Delta S \approx 16\pi \Pi'_{33}(q)|_{q^2=0}$ [11]. As we will see, this contribution can be neglected.

The $\Pi$'s contribute to $S$ and $T$, but we postpone a detailed discussion of the model's prediction for electroweak precision measurements, since, as discussed in section 3 additional contributions to $S$ and $T$ arise from the rescaling of gauge bosons. A careful treatment of the fermionic sector is necessary.

# 5 Fermionic operators

The coefficients of the operators in Eq. (3.3) get contributions from integrating out KK modes of gauge fields (at tree-level) as shown in the Feynman diagrams of Figs. 2 and 3. The Feynman diagram in Fig. 2 is evaluated by integrating fermion zero-mode wavefunction with propagator from $z$ to TeV brane (including metric/fünfbein factors). This gives the fermion-Higgs higher-dimensional operator in Eq. (3.3) with coefficients (up to $O\left(z_h^2/z_v^2\right)$) as follows[5]. From exchange of KK modes of $W_L$, we get (see appendix C for details)

$$\begin{aligned} x(c) &= 16\pi^2 v^2 g_5^2 \int dz \sqrt{G}\, \frac{z}{z_h}\, \chi_0^2(c,z) \left(G^{5D}_{q=0(++)}(z,z_v) - G^{(0)}_{q(++)}\right) \\ &= g^2\left(16\pi^2 v^2 z_v^2\right)\big[\frac{1}{4}\left(1-\frac{1}{k\pi r_c}\right) + \frac{1}{1-e^{k\pi r_c(2c-1)}}\frac{1-2c}{3-2c}\left(-\frac{k\pi r_c}{2}+\frac{5-2c}{4(3-2c)}\right)\big], \end{aligned} \tag{5.1}$$

From exchange of KK modes of hypercharge and $Z'$

$$\begin{aligned} y(c) &= 16\pi^2 v^2 g_5'^2 Y Y^H \int dz \sqrt{G}\, \frac{z}{z_h}\, \chi_0^2(c,z) \left(G^{5D}_{q=0(++)}(z,z_v) - G^{(0)}_{q(++)}\right) \\ &\quad + 16\pi^2 v^2 g^2_{Z'\,5} Q_{Z'} Q^H_{Z'} \int dz \sqrt{G}\, \frac{z}{z_h}\, \chi_0^2(c,z) \left(G^{5D}_{q=0(++)} - G^{(0)}_q\right)(z,z_v) \\ &= g'^2\left(16\pi^2 v^2 z_v^2\right) Y Y^H \big[\frac{1}{4}\left(1-\frac{1}{k\pi r_c}\right) + \frac{1}{1-e^{k\pi r_c(2c-1)}}\frac{1-2c}{3-2c}\left(-\frac{k\pi r_c}{2}+\frac{5-2c}{4(3-2c)}\right)\big] \\ &\quad + g^2_{Z'} Q_{Z'} Q^H_{Z'}\left(16\pi^2 v^2 z_v^2\right)\frac{1}{1-e^{k\pi r_c(2c-1)}}\frac{1-2c}{3-2c}\left(-\frac{k\pi r_c}{2}\right) \end{aligned} \tag{5.2}$$

[5]See also references [7, 9, 10, 14] for discussion of this effect in a different language.

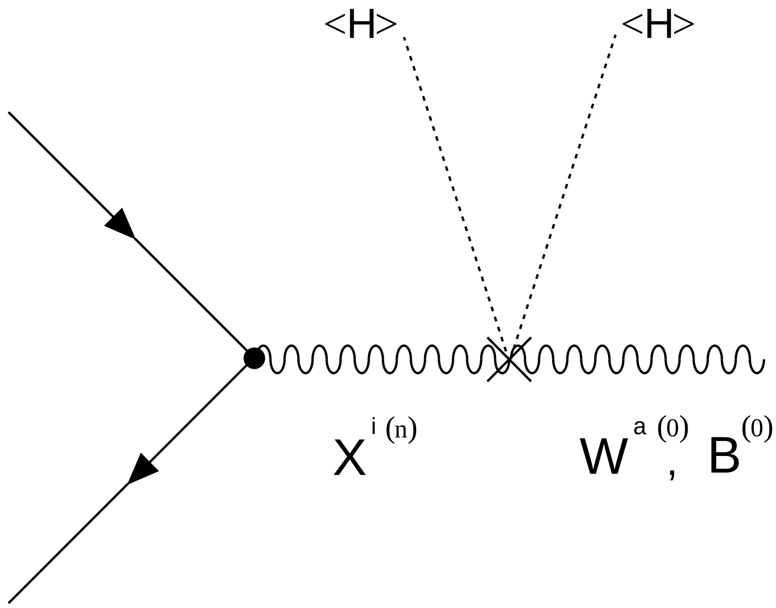


Figure 2: Contribution to fermion-Higgs operator. As in appendix B, $X^i$ is any field coupling to the $W^a$'s.

The factor of $z/z_h$ (inside $z$ integral) in first line is from fünfbein. $Q_{Z'}$ and $Q^H_{Z'}$ denotes the $Z'$ charge of fermion and Higgs (see Eq. (2.15).

A similar computation of the Feynman diagram in Fig. 3 gives coefficient of "compositeness" operator in Eq. (3.3), $\bar{\psi}\tau^a\psi\bar{\psi}'\tau_a\psi'$ (from exchange of KK modes of $W$):

$$\begin{aligned} V(c,c') &= 16\pi^2 v^2 g_5^2 \int dz \sqrt{G(z)}\,\frac{z}{z_h}\,\psi^{(0)\,2}(c,z)\int dz'\sqrt{G(z')}\,\frac{z'}{z_h}\,\psi^{(0)\,2}(c',z') \\ &\quad \times \left(G^{5D}_{q=0(++)}(u,v) - G^{(0)}_q\right) \end{aligned} \tag{5.3}$$

We obtain $V \approx g^2\; 16\pi^2 v^2 z_v^2/(4k\pi r_c)$ for $c, c' > 1/2+\epsilon$ (as applicable to light fermions): this coefficient is negligible for $z_v^{-1} \gtrsim$ TeV and similarly for exchange of hypercharge or $Z'$ KK modes[6]. See also references [12, 13].

## 5.1 Light fermions

If light fermions are at $c > 1/2+\epsilon$ (such that $e^{k\pi r_c\epsilon} \gg 1$: $\epsilon \gtrsim 0.1$ suffices) in order to address flavor (as mentioned before [12, 16]), then second term proportional to $Y$ in Eq. (5.2) can be

[6]Coefficient of the operator (light fermion)$^2$ (top or *left*-handed bottom)$^2$ will be larger since $c$ for top quark or *left-handed* bottom $< 1/2$ (see section 5.2), but it plays no role in fit to precision electroweak data, although it will affect, say, $e^+e^- \to \bar{b}_L b_L$ at high-energy colliders.

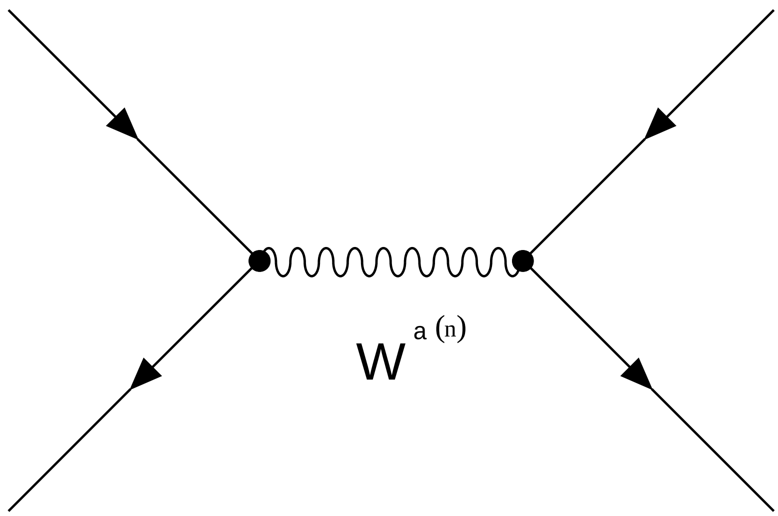


Figure 3: Contribution to $\psi^4$.

neglected. Also, KK modes of $Z'$ couple very weakly to light fermions and so there are no operators proportional to $Z'$ charge (i.e., last term in Eq. (5.2)). Thus, coefficients $x$ and $y$ of these fermion-Higgs operators are of the special form discussed in section 3 with

$$a \approx 4\pi^2 v^2 z_v^2. \tag{5.4}$$

Hence, the $S$ parameter in our model is given by (see Eq. (3.7))

$$\begin{aligned} S &\approx 2\pi v^2 z_v^2 \\ &\approx 2\pi v^2 \frac{6}{m_{gauge}^{(1)\,2}}, \end{aligned} \tag{5.5}$$

where $m_{gauge}^{(1)}$ is the mass of the lightest KK mode of gauge boson (see Eq. (D.2)). Here we have neglected $S$ from gauge-Higgs sector since it is of higher order in $z_v v$ (see section 4.2).

There is an interesting possibility that we will not pursue here where contribution to $S$ parameter arising from the fermion-Higgs operators in Eq. (3.3) is suppressed completely for $c = 1/2$ as can be seen from Eqs. (5.1) and (5.2). However, in order to fit observed light fermion masses with $c = 1/2$, we would have to introduce very small dimensionless numbers into our fundamental theory. While this is radiatively stable, it goes against the general philosophy adopted in this paper.

Similarly, the $T$ parameter is given by (using Eqs. (3.7) and (4.9))

$$\begin{aligned} T &\approx \big[\frac{\pi}{2}\frac{\tilde{g}^2}{e^2} v^2 z_v^2 k\pi r_c\big]\frac{\tilde{M}^2}{4k^2} \\ &\approx \big[3\pi\ k\pi r_c \frac{v^2}{m_{gauge}^{(1)\,2}}\frac{\tilde{g}^2}{e^2}\big]\frac{\tilde{M}^2}{4k^2}, \end{aligned} \tag{5.6}$$

where the quantity in [...]'s is the $T$ parameter (assuming $\tilde{g} = g'$) that would result if we repeated our analysis with *purely* SM gauge group in bulk rather than our present extended gauge sector.

## 5.2 Top and bottom

We can obtain the $4D$ Yukawa coupling $\lambda$ in terms of the $5D$ Yukawa coupling $\lambda_5$ (see, for example, [12]):

$$\lambda = \lambda_5 k \frac{\sqrt{|(1-2c_L)(1-2c_R)|}}{\left(1-e^{k\pi r_c(2c_L-1)}\right)\left(1-e^{k\pi r_c(2c_R-1)}\right)} \tag{5.7}$$

The quick argument for choosing $c_{L,R} < 1/2$ for top quark is that, for $c_L$ (or $c_R$) $> 1/2$, the $4D$ Yukawa coupling is (exponentially) suppressed (see Eq. (5.7)) and hence we consider $c_{L,R} < 1/2$ for top quark to obtain $\lambda_t \sim 1$.

For $c_{L,R} < 1/2 - \epsilon$, we get

$$\lambda_t \approx \lambda_{t\,5} k \sqrt{(1-2c_L)(1-2c_R)} \tag{5.8}$$

Since coefficient in Eq. (5.2) is different for $b_L$ than for light fermions, the effect on coupling of $b_L$ to $Z$ arising from the operators in Eq. (3.3) can*not* be redefined into $S$ (see discussion in section 3) and must be treated separately:

$$\begin{aligned}
\frac{\delta\left(g_Z^{b_L}\right)}{g_Z^{b_L}} &\approx m_Z^2 z_v^2 \left( \left[1 + \frac{g_{Z'}^2 Q_{Z'} Q_{Z'}^H}{g_Z^2 Q_Z Q_Z^H}\right] \frac{1-2c}{3-2c} \left( -\frac{k\pi r_c}{2} + \frac{5-2c}{4(3-2c)} \right) \right. \\
&\approx \frac{m_Z^2}{\left(0.4 m_{gauge}^{(1)}\right)^2} \left( -\frac{k\pi r_c}{2} + O(1) \right) 0.9 \, \frac{1-2c}{3-2c}
\end{aligned} \tag{5.9}$$

using the $Z'$ charges of Higgs and $b_L$. Here, $Q_Z = \tau_L^3 - Q_{em}\sin^2\theta_W$ and $Q_Z^H$ are $Z$ charges of $b_L$ and Higgs.

To obtain $m_b \ll m_t$ with*out* hierarchy in $5D$ Yukawa coupling ($c_L$ is *same* for top and bottom), we choose $c$ for $b_R > 1/2$.

# 6 $T$ at loop level in Scenario II

An interesting case to consider is when $SU(2)_R$ is unbroken in the bulk (our Scenario II), $\tilde{M} = 0$. In that case, remarkably, bulk custodial isospin symmetry forces loop contributions to $T$ parameter to be UV finite (and hence calculable) and these are the dominant contribution

to $T$. This is because contribution to $T$ requires both electroweak symmetry breaking on IR brane and $SU(2)_R$ breaking which is localized on UV brane. For remainder of this section, we will consider this case.

Because custodial-isospin violation is due to breaking of $SU(2)_R$ by boundary condition on Planck brane, there is *no* zero-mode for $\tilde{W}^{\pm}$ and KK spectrum is different for $\tilde{W}^{\pm}$ and $\tilde{W}^3$ (see appendix D). Similarly, there is *no* zero-mode for $d'_R$ and KK spectrum can be different for $u_R$ and $d'_R$ (as can be seen from appendices E and F). Hence, loop diagrams will have to involve *right*-handed $\tilde{W}$ and/or $t$ (and other fermions) in order to give $T$.

An example of a Feynman diagram with $\tilde{W}$ zero and KK modes, but with*out* fermions is shown in Fig. 4. This diagram with $\tilde{W}^3$ *zero*-mode gives $W^{\pm}W^{\mp}$ mass term $\sim g^2 \frac{g'^2 g^2 2k\pi r_c}{16\pi^2} \frac{v^4}{m_{gauge}^{(1)\,2}}$. A brief explanation is as follows: the quartic $W$ vertex is $g^2$ since external legs are *zero*-modes and each Higgs vertex gives $g' g\sqrt{2k\pi r_c}$, where $g'$ is due to $\tilde{W}^3$ (or hypercharge) *zero*-mode propagator and $g\sqrt{2k\pi r_c}$ is coupling of $W$ KK mode to Higgs on TeV brane (see appendix D). This diagram does not give $W^3W^3$ mass term since there is no quartic $W^3$ coupling. We can estimate contribution with *KK* modes of $\tilde{W}^{\pm,3}$ as follows: KK modes of $\tilde{W}^{\pm}$ and $\tilde{W}^3$ are split at $O\left(1/\left[k\pi r_c\right]\right)$ (see appendix D), whereas their coupling to Higgs is enhanced by $\sim \sqrt{k\pi r_c}$ (compared to $\tilde{W}$ zero-mode) to so that KK contribution (to both $W^{\pm}W^{\mp}$ and $W_3W_3$ mass terms) is comparable to that of $\tilde{W}^3$ zero-mode.

The Feynman diagrams with $t_R$ (and other *right*-handed fermions) are shown in Figs. 5 and 6.

Let us estimate the Feynman diagram with*out* Yukawa insertion (Fig. 5). The contribution of $t_R^{(0)}$ gives $\Pi \sim \left(\frac{1}{2}\right)^2 \frac{3}{16\pi^2}\tilde{g}^4 \left(\sqrt{2k\pi r_c}\right)^4 \frac{(v/2)^4}{m_{gauge}^{(1)\,2}}$, where the factors of $1/2$ are from quantum number of $t_R$ and Higgs and the coupling of $t_R$ zero-mode (and Higgs) to $\tilde{W}^3$ *KK* mode is enhanced by $\approx \sqrt{2k\pi r_c}$ (compared to $\tilde{W}^3$ zero-mode) since $t_R^{(0)}$ is localized near TeV brane. The fractional mass splitting of *KK* modes of $t_R$ and $b'_R$ can be $O(1)$ depending on $c_R$ (see appendices E and F), while their couplings to KK modes of $\tilde{W}$ are almost same as that of $t_R^{(0)}$ since they are also localized near TeV brane. Hence, contribution of KK modes can be comparable to that of $t_R^{(0)}$.

As we will see, the Feynman diagram involving insertions of *top* Yukawa coupling, Fig. 6 dominates over the diagram with*out* top Yukawa insertion (Fig. 5) (and diagrams with other fermions) and also over the diagram with*out* fermions (Fig.4). So, we concentrate on the diagram in Fig.6.

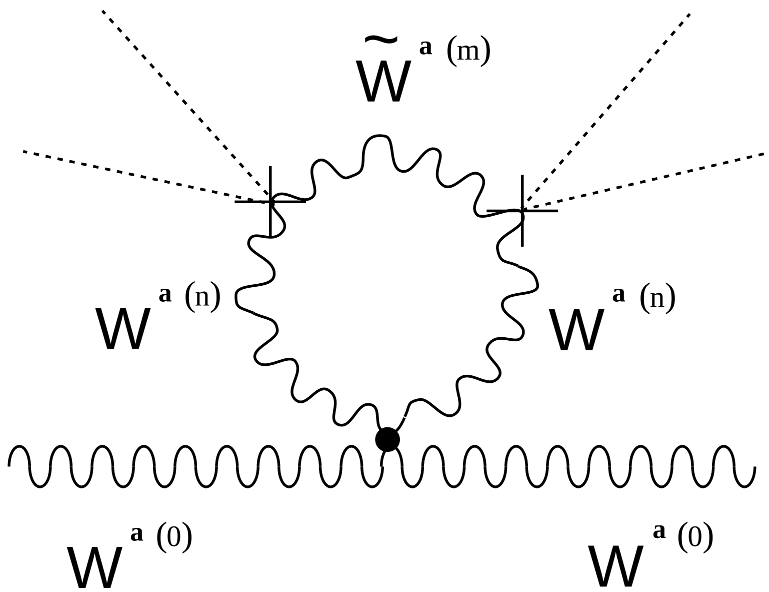


Figure 4: Contribution to $T$ from gauge loop

## 6.1 $T$ from top quark KK modes

To calculate the diagram in Fig. 6, we need (a) spectrum of KK modes of *left*-handed top and bottom and also $t_R$ and its $SU(2)_R$ partner, $b'_R$ (*not* the physical $b_R$) and (b) their couplings to Higgs (or the $t_L^{(m)} t_R^{(n)}$ and $b_L^{(m)} b_R'^{(n)}$ "mass insertions"). The spectrum and couplings to Higgs are calculated in the appendices E and F to which the reader is referred for details. We find it *not* convenient to convert sum over KK modes into propagators (unlike before) and so we will work directly in terms of KK modes.

We begin with $c_R < -1/2 - \epsilon$, where $\epsilon \gtrsim 0.1$. In this case, we get a "very light" (much lighter than $z_v^{-1}$) $b'_R$ mode. This is ruled out experimentally.

For a slightly larger $c_R$, namely, $c = -1/2 + \epsilon$ (with $\epsilon \sim 0.1$), we can show that the lightest $b'_R$ mode is *not* lighter than $z_v^{-1}$ (and hence *not* ruled out experimentally unlike before), but its mass is smaller than that of the lightest $(t, b)_L$ and $t_R$ KK mode. Other modes of $b'_R$ are almost degenerate with KK modes of $t_R$. Also, because $c_R \approx 1/2$, mass insertions in Fig. 6 involving $t_R$ and $b'_R$ *KK* modes are the *same* as those involving the $t_R$ *zero*-mode. One can show that this results in small $T$.

Next, we consider $c_R \sim 0$. It is clear that *all* modes of $t_R$ and $b'_R$ need to be considered since KK spectrum is different for $t_R$ and $b'_R$. The $t_L^{(n\neq0)} t_R^{(0)}$ mass insertion is $m_t f(c_L)$, whereas the $t_L^{(m\neq0)} t_R^{(n\neq0)}$ and $b_L^{(m\neq0)} b_R'^{(n)}$ mass insertions are $m_t f(c_L) f(c_R)$, where $f(c)$ is given in Eq. (E.8). For any given *KK* modes of $t_L$ and $t_R$ in the loop, the diagram in Fig. 6 gives (with *four*

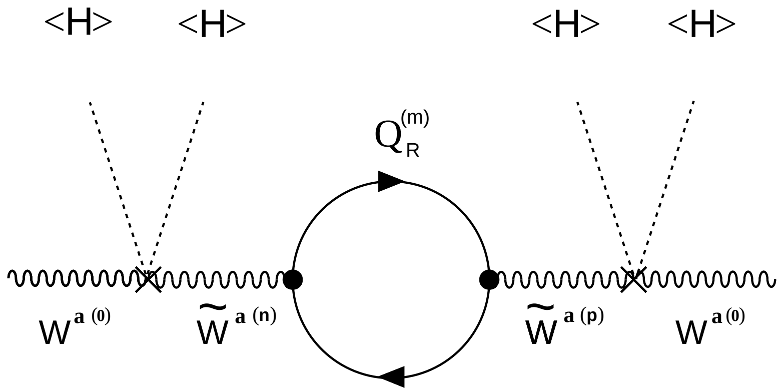


Figure 5: Contribution to $T$ from top loop with*out* Yukawa insertion.

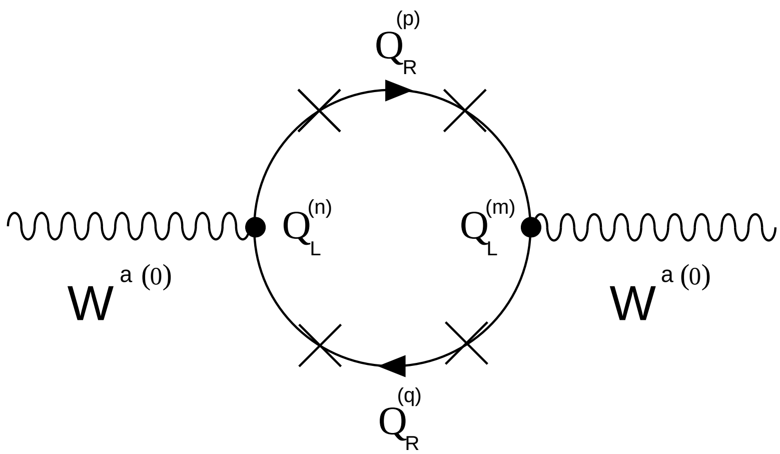


Figure 6: Contribution to $T$ from top loop with Yukawa insertion. The "$\times$" denotes a $t_L^{(m)} t_R^{(n)}$ or $b_L^{(m)} b_R^{\prime\,(n)}$ "mass insertion".

insertions of $t_L^{(m\neq 0)} t_R^{(n\neq 0)}$)

$$\begin{aligned}
\Pi_{33}(0)(m,n,p,q) \;&\approx\; \frac{-3}{64\pi^2}\frac{f(c_L)^4 f(c_R)^4 m_t^4}{m^2_{t_L^{(m)}}} \\
&\times \int_0^\infty dx \frac{x^2\left[x^2+2\left(1+\frac{m^2_{t_L^{(n)}}}{m^2_{t_L^{(m)}}}\right)x+\frac{m^2_{t_L^{(n)}}}{m^2_{t_L^{(m)}}}\right]}{(x+1)^2\left(x+\frac{m^2_{t_L^{(n)}}}{m^2_{t_L^{(m)}}}\right)^2\left(x+\frac{m^2_{t_R^{(p)}}}{m^2_{t_L^{(m)}}}\right)\left(x+\frac{m^2_{t_R^{(q)}}}{m^2_{t_L^{(m)}}}\right)}
\end{aligned} \tag{6.1}$$

There are similar contributions to $\Pi_{33}$ involving $KK$ modes of $b_L$ and $b'_R$ and to $\Pi_{11}$ involving $KK$ modes of $(t,b)_L$ and $t_R$, $b'_R$. For a $t_R$ *zero*-mode in the loop, we set $f(c_R)$ to 1. The spectrum

is not very sensitive to $c_L$ and $c_R$, so we use spectrum for $c_L = 1/2$ since we will choose $c_L \sim 0.4$, i.e., close to 1/2 in section 7 and $c_R = 0$:

$$\begin{aligned} m_{t_R^{(n)}} z_v & \approx \text{zeroes of } J_{\frac{1}{2}} & = n\pi \\ m_{b_R^{\prime (n)}} z_v & \approx \text{zeroes of } J_{-\frac{1}{2}} & = \left( n - \frac{1}{2} \right) \pi \end{aligned} \tag{6.2}$$

and

$$\begin{aligned} m_{t_L^{(n)}, \, b_L^{(n)}} z_v & \approx \text{zeroes of } J_0 \\ & \approx \left( n - \frac{1}{4} \right) \pi \end{aligned} \tag{6.3}$$

$f(c_L)$ in Eq. (6.1) is sensitive to $c_L$ (see Eq. (E.8)) since we will choose $c_L \sim 0.4$ (i.e., close to 1/2) in section 7, but the dependence on $f(c_L)$ is analytic. Whereas, $f(c_R)$ is *not* very sensitive to $c_R$ since $c_R \sim 0$ (see Eq. (E.8)) and so we set $f(c_R) = f(0) = \sqrt{2}$ in Eq. (6.1).

A numerical evaluation of the loop integral and the sum over *KK* modes of $(t, b)_L$ (i.e., $m, n$) and *all* modes of $t_R$, $b'_R$ (i.e., $p, q$) in Eq. (6.1) gives (as mentioned earlier, the sum over modes converges because of bulk $SU(2)_R$ gauge symmetry)

$$\begin{aligned} T_{\text{KK top}} & = \frac{4\pi}{\sin^2 \theta_W \cos^2 \theta_W M_Z^2} \left[ \Pi_{11}(0) - \Pi_{33}(0) \right] \\ & \approx T_{\text{SM top}} \left( \frac{f^2(c_L) m_t}{m_{t_L^{(1)}}} \right)^2 0.7, \end{aligned} \tag{6.4}$$

where 0.7 is from (numerical) integration and KK sum, $m_{t_L^{(1)}}$ is the mass of the lightest $(t, b)_L$ KK mode and $T_{\text{SM top}} = \frac{3}{16\pi \sin^2_{\theta_W} \cos^2_{\theta_W}} \frac{m_t^2}{m_Z^2} \approx 1.2$.

If we replace KK modes of *left*-handed top (or bottom) by *zero*-mode in this Feynman diagram, then we lose a factor of $f(c_L) \gg 1$ and so such effects are sub-leading. Of course, Fig. 6 will *only* zero-modes in the loop is the SM top quark contribution to $T$.

Now, we can see why $T$ from other (light) fermion KK modes is smaller than that from top quark. The diagram in Fig. 6 with *zero*-mode of, say, $u_R$ is smaller than in the case of top quark (even if $\lambda_5$ is comparable for all fermions) due to smaller mass insertion which, in turn, is due to larger $f(c_R)$ for light fermions. Whereas, for KK modes of $u_R$, mass insertions can be *comparable* to that in the case of top quark (if $\lambda_5$ is comparable for all fermions), but the non-degeneracy of $u_R$ and $d'_R$ KK modes is smaller than in the case of top quark, again due to larger $f(c_R)$ for light fermions. Hence, the contribution of KK modes of $u_R$ is also smaller than in the case of top quark.

# 7 Electroweak fit

We now put the pieces together and fit the precision electroweak data.

## 7.1 $Z \to \bar{b}_L b_L$

From Eq. (5.9), we see that, for $c_L \stackrel{>}{\sim} 0.3$ and $m_{gauge}^{(1)} \stackrel{<}{\sim} 4$ TeV, the shift in the coupling of $b_L$ to $Z$ is $\stackrel{<}{\sim} 1\%$ which is allowed by precision electroweak data. If $c_L \stackrel{<}{\sim} 0.3$, then $m_{gauge}^{(1)} \gg$ few TeV to be consistent with $Z \to \bar{b}_L b_L$, a case not of interest to us here. Hence, we choose $c_L \stackrel{>}{\sim} 0.3$.

## 7.2 $S$ and required $T_{RS}$

We choose $c > 1/2$ for light fermions in order to address flavor issues: hierarchy of fermion masses and suppression of FCNC's as mentioned before [12, 16]. From Eq. (5.5), this gives

$$S_{RS} \sim 0.2 \text{ for } m_{gauge}^{(1)} \sim 3-4 \text{ TeV}. \tag{7.1}$$

A smaller $m_{gauge}^{(1)}$ will give too large $S$ (unless we choose $c = 1/2$ for light fermions as we discussed in section 5.1) such that we cannot fit data (independent of $T$) So, we will not consider $m_{gauge}^{(1)} < 3$ TeV.

A heavy Higgs, say, $m_H \sim 500$ GeV gives [30]

$$\begin{aligned} S_{SM} &\sim +0.1 \\ T_{SM} &\sim -0.15, \end{aligned} \tag{7.2}$$

where $S_{SM}$ is measured relative to SM with Higgs mass 100 GeV. Adding $S_{SM}$ to $S_{RS} \sim +0.2$ shows that

$$T_{RS}^{required} \sim +0.35 - 0.5 \tag{7.3}$$

is required to get a reasonable fit to data [30].

For smaller Higgs mass, say, $\sim 200$ GeV we have [30]

$$\begin{aligned} S_{SM} &\sim +0.05 \\ T_{SM} &\sim -0.05 \end{aligned} \tag{7.4}$$

so that (adding it to $S_{RS} \sim +0.2$) shows

$$T_{RS}^{required} \sim +0.15 - 0.35 \tag{7.5}$$

is required to fit data [30].

### 7.3 Scenario I: $\tilde{M} \neq 0$

From Eq. (5.6), we see that the *tree*-level value of $T$ (*after* fermion-Higgs operators are transformed into $S$ as discussed in section 3) is controlled by $\tilde{M}/k$. The fractional splitting of the KK masses of $\tilde{W}^{\pm}$ and $\tilde{W}^3$ is given by $\sim \tilde{M}^2/\left(4k^2\right)$ (see appendix D) so that, assuming that the fractional mass splitting $< 1/2$ to warrant approximation of small custodial-isospin breaking

$$
\begin{aligned}
T_{RS} &\sim 2 \times \frac{\tilde{M}^2}{4k^2} \text{ for } m^{(1)}_{gauge} \sim 3-4 \text{ TeV} \\
&\sim 0-1,
\end{aligned} \tag{7.6}
$$

where we have assumed $\tilde{g} \sim g'$. From Eqs. (7.6), (7.3) and (7.5), our model can fit the data for both light and heavy Higgs using control parameter $\tilde{M}/k$ for a sizable portion of its range.

### 7.4 Scenario II: $\tilde{M} = 0$

In the absence of bulk $SU(2)_R$ breaking, it is interesting to see if the required $T$ can be generated by radiative effects. We saw in section 6 that $T$ from top loop in Fig. 6 dominates and depends on $c_L$ and $c_R$ for top quark. $T$ is small for $c_R \sim -1/2$. From Eq. (6.4), for $c_R \approx 0$, $0.3 \lesssim c_L \lesssim 0.4$ and gauge/$t_L$ KK masses $\sim 3-4$ TeV, we get

$$
T_{\text{KK top}} \sim 0.04 - 0.3 \tag{7.7}
$$

From Eqs. (7.7), (7.3) and (7.5), we see that with radiatively generated $T$, the RS model agrees with the electroweak data for a sizable portion of the allowed range of $T$ for light Higgs, whereas fit to data with heavy Higgs is possible only at the upper limit of the allowed range of $T$.

We have not considered $c_L > 0.4$, since then the theory is no longer weakly coupled at the KK mass scale, the strong coupling scale (from loop corrections to Higgs couplings due to top loop) estimated as:

$$
\begin{aligned}
\Lambda_t &\sim z_v^{-1} \frac{4\pi}{\lambda_{t\,5} k} \\
&\sim 4 m_{t^{(1)}} \sqrt{(1-2c_L)(1-2c_R)}
\end{aligned} \tag{7.8}
$$

Similarly, to keep the theory weakly coupled we have chosen $c_R \lesssim 0$ (given that $c_L \gtrsim 0.3$ to avoid excessive corrections to bottom couplings). See section 10 for further discussion.

# 8 Collider signals

Our model has rather distinctive phenomenology as follows. As discussed earlier, the Higgs couplings to electroweak gauge KK modes are enhanced (compared to that of *zero*-modes) by

$\sim \sqrt{k\pi r_c}$ as expected from their CFT dual interpretation as strongly coupled composites (see section 9). Thus, *longitudinal* $W, Z$ (eaten Higgs component) fusion into electroweak gauge KK modes (with masses $\sim$ few TeV) is enhanced. In turn, these KK modes have sizable decay widths to *longitudinal* $W/Z$'s:

$$
\begin{aligned}
W_{long.}\, Z_{long.} \text{ and } W_{long.}\, W_{long.} & \stackrel{g\sqrt{k\pi r_c}}{\longrightarrow} W^{\pm\,(n)}, Z^{(n)}, \tilde{W}^{\pm\,(n)}, Z'^{\,(n)} \\
& \stackrel{g\sqrt{k\pi r_c}}{\longrightarrow} W_{long.}\, Z_{long.} \text{ and } W_{long.}\, W_{long.}
\end{aligned} \tag{8.1}
$$

Note that considerably below the energies of these resonances in longitudinal $W/Z$ scattering, the growth of the cross section is softened by Higgs exchange.

There are also unique signals involving fermion modes. For example, $t_R$ is strongly coupled to gluon and *right*-handed $\tilde{W}$ KK modes since its wavefunction is localized near TeV brane, leading to $k\pi r_c$-enhanced production of gluon and $\tilde{W}$ KK modes through gluon fusion via $t_R$ *loop*. Conversely, gluon and *right*-handed $\tilde{W}$ KK modes have strong decays to $t_R$:

$$
\begin{aligned}
\text{gluon} + \text{gluon} & \stackrel{\text{top loop}}{\longrightarrow} \text{gluon}^{(n)}, Z^{(n)}, Z'^{\,(n)} \\
& \stackrel{g\sqrt{k\pi r_c}}{\longrightarrow} \bar{t}_R t_R
\end{aligned} \tag{8.2}
$$

Again, this is expected since in the dual CFT interpretation (see section 9) $t_R$ has a large admixture of CFT composites.

Another interesting signal arises from enhanced Higgs-$t_R^{(0)}$-$b_L^{(n)}$ coupling $\sim \lambda_t f(c_L) \sim \sqrt{10}$ (for $c_L \sim 0.4$), leading to $b_L^{(n)}$ production by *longitudinal* $W$-$t_R^{(0)}$ fusion:

$$
t_R\, W_{long.} \stackrel{f(c_L)\lambda_t}{\rightarrow} b_L^{(n)}. \tag{8.3}
$$

# 9 CFT interpretation

Our model is dual [4], in the sense of the AdS/CFT correspondence [3], to a strongly coupled large-$N$ $4D$ CFT with $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ global symmetry whose $SU(3)_c \times SU(2)_L \times U(1)_Y$ subgroup is gauged. Higgs on TeV brane corresponds to a composite of the CFT responsible for spontaneous breaking of $SU(2)_L \times SU(2)_R$ symmetry. That is, our model is dual to a particular type of a composite Higgs model. In the dual interpretation, the hierarchy problem is solved by this compositeness as opposed to any symmetry.

In the dual picture, the $S$ parameter arises from exchange of spin-1 CFT composites ("techni-$\rho$'s") for which one would naively expect $S \sim 16\pi v^2/m_\rho^2$ [11] – this agrees roughly with Eq. (5.5). However, because we have a Higgs in our model, $m_\rho$ is *not* tied to the weak scale and can be made heavy enough to adequately suppress the $S$ parameter.

The dual interpretation of light fermions with $c > 1/2$ is as follows. We have fundamental fermions external to the CFT, coupled to *ir*relevant fermionic CFT operators so that the mixing of the external fermion with the CFT composites is small, i.e., the resulting massless state which is the SM fermion is mostly fundamental. Yukawa couplings to composite Higgs must go through this small mixing and are therefore also small. Also, because SM fermions are mostly fundamental with small mixing to CFT, higher-dimensional fermion-*Higgs* operators as in Eq. (3.3) are highly suppressed for light fermions. The small mixing with CFT sector also naturally suppresses unwanted FCNC's.

For the third generation fermions, one can therefore see a tension as follows. To obtain $\lambda_t \sim 1$, it is clear that the top should couple to a relevant CFT operator (dual to $c < 1/2$), i.e, fundamental top quark should mix appreciably with CFT composites in order for the SM top to have $O(1)$ coupling to the composite Higgs. However, if CFT operator coupled to fundamental *left*-handed top and hence $b_L$ is *relevant*, then the fundamental $b_L$ mixes substantially with the CFT composites and induces higher-dimensional bottom-Higgs operator in Eq. (3.3) contributing to $Z \to \bar{b}_L b_L$. To be consistent with $Z \to \bar{b}_L b_L$ data, $b_L$ coupling to CFT operator can be at most *mildly* relevant. But, then, in order to get the top Yukawa coupling, fundamental $t_R$ has to couple to a more relevant operator. That is, the SM $t_R$ must contain sizable admixture of CFT composites. This mechanism for generating Yukawa hierarchies is similar to the proposal of reference [31], but translated into a CFT context here.

The central feature of the dual CFT that suppresses the $T$ parameter is exact (in scenario II) custodial isospin from $SU(2)_R$ symmetry. The dual interpretation of breaking of $SU(2)_R$ by UV brane boundary condition is that this custodial isospin symmetry is *not* fully gauged. Rather, it is a symmetry of the CFT Higgs sector when isolated from fundamental fermions and gauge fields. These fundamental fermions and their couplings to the CFT explicitly break global $SU(2)_R$.

Of these custodial-isospin violating effects due to fundamental fields, the largest is due to coupling of fundamental $t_R$ to *just* a isospin component of a moderately *relevant* CFT operator. As a result, the CFT flows to a *new* fixed point which *violates* custodial-isospin at *sub*-leading order in $N$. This results in suppression of the $T$ parameter by $1/N$ in scenario II, dual to generating $T$ at *loop* order in the RS model.

The dual of scenario I with small bulk breaking of $SU(2)_R$ is that the CFT Higgs sector has *approximate* custodial isospin (global) symmetry. Therefore, the $T$ parameter is suppressed, but non-zero and can dominate over top-induced contribution discussed above.

## 10 Discussion and Outlook

We have studied the precision electroweak fit in a RS1 scenario with gauge fields and fermions in the bulk. In order to soften the $T$-parameter constraints we have enhanced the electroweak gauge structure to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, recovering the usual gauge group via Planck brane boundary conditions and Higgsing. This model has a natural exact (approximate) custodial isospin *gauge* symmetry in the *bulk* that renders the $T$-parameter zero (small) at leading order in our weak/KK-scale expansion, compared with the excessive values obtained in earlier studies without bulk custodial isospin. Localizing the light fermions near the Planck brane decreases the $S$-parameter so the electroweak fit is possible for KK masses of a few TeV. This type of bulk localization also allows us to incorporate the attractive mechanism for generating flavor structure of Refs. [12] [16].

We have then complete and realistic models without any large hierarchies among input parameters. The collider signals of our model are also quite distinctive and arise from the enhanced couplings of the top quark and longitudinal $W$'s and $Z$'s to KK modes.

Important insight into our model comes in the light of the AdS/CFT correspondence. Our model is dual to a strongly coupled CFT Higgs sector enjoying a custodial *global* symmetry, of which the minimal Higgs is a composite arising after conformal invariance is broken, with couplings to fundamental SM gauge and fermion fields which necessarily violate custodial isospin. The large top mass is correlated with the right-handed top having a large admixture of a composite fermion within it, affecting its couplings.

The strong coupling scale (especially in our Scenario II) in the top-Higgs sector is not much larger than the scale of the first KK masses. The interpretation of this in effective field theory is that above this scale the Higgs-top physics is sensitive to the detailed structure of the IR brane, which here we treated as point-like in the extra dimension. However this does not affect the key results of our paper because bulk couplings have a higher strong-coupling scale. Nevertheless it would be interesting in the future to introduce explicit brane size and structure (and therefore structure in the Higgs system) within RS effective field theory so as to raise the scale of strong coupling on the IR brane to that of the bulk. This appears quite feasible.

Another aspect of our scenario is the remnant fine tuning needed in order to get a light Higgs compared to KK masses. It is important to note that there are two contributions to the Higgs mass at loop level, one coming from the zero modes and another from KK modes. The latter contribution dominates since the KK couplings are enhanced over the zero mode couplings. We estimate fine tuning to be of the order of 1% in mass-squared from the top-Higgs couplings. We suspect the tuning can be made much milder by introducing symmetry protection for the Higgs from KK modes (as opposed to zero-modes which is inescapable). In particular the ideas

of references [33] and [32] seem promising and attractive.

Finally one can ask whether our models can be embedded into a GUT theory along the lines proposed in reference [20]. Since we have enlarged the gauge structure, minimal $SU(5)$ will not work as in reference [20], but both $SU(2)_R$ and $SU(5)$ can be easily embedded into $SO(10)$, and a realistic model incorporating both symmetries seems feasible.

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## Appendix

## A 5D Gauge Propagators

We use $5D$ mixed position-momentum space propagators in Feynman gauge. The general propagator from $z$ to $z'$ for $(+,+)$ boundary conditions with momentum $p$ is (up to tensor structure) [18]:

$$\begin{aligned} G^{5D}_{p(++)}(u,v) &= \frac{\pi}{2}\frac{kuv}{\big[-Y_0(pz_h)J_0(pz_v)+J_0(pz_h)Y_0(pz_v)\big]} \\ &\quad\times\big[-Y_0(pz_h)J_1(pu)+J_0(pz_h)Y_1(pu)\big]\big[-Y_0(pz_v)J_1(pv)+J_0(pz_v)Y_1(pv)\big], \end{aligned} \tag{A.1}$$

where $u = min(z,z')$ and $v = max(z,z')$.

For $(-,+)$ boundary condition with bulk mass $M$, the propagator is

$$\begin{aligned}
G^{5D}_{p(-+)}(u,v,M) &= \frac{\pi}{2}\frac{kuv}{\big[-Y_\nu(pz_h)\tilde{J}_\nu(pz_v)-J_\nu(pz_h)\tilde{Y}_\nu(pz_v)\big]} \\
&\quad\times\big[-Y_\nu(pz_h)J_\nu(pu)+J_\nu(pz_h)Y_\nu(pu)\big]\big[-\tilde{Y}_\nu(pz_v)J_\nu(pv)+\tilde{J}_\nu(pz_v)Y_\nu(pv)\big],
\end{aligned} \tag{A.2}$$

where $\nu=\sqrt{1+M^2/k^2}$ and $\tilde{J}_\nu(x)\equiv(1-\nu)/x\ J_\nu(x)+J_{\nu-1}(x)$.

# B KK Sum to 5D Propagator Conversion

We expand corrections to the zero mode gauge propagators in $vz_v$ (see section 4), i.e., we work in the insertion approximation for the Higgs vev (see discussion at the end of section 3). We isolate from $\mathcal{L}_{\rm IR}$ all terms containing only zero modes. The zero modes reproduce electroweak symmetry breaking at order $v^2$. In particular, the zero modes yield $M_W^2=M_Z^2\cos^2\theta_W$, where $\theta_W$ is the weak mixing angle.

As in the standard framework for oblique corrections [11], we will consider diagrams with $W^{a\,(0)}$'s on the external lines. The Higgs vev mixes $W^{a\,(0)}$ with its own KK modes, and also the $\widetilde{W}^a$, $Z'$ and $B$ mass eigenstates. The KK expansion for the dimension $\frac{3}{2}$ field is

$$W^{a\,5D}_\mu(x,y)=\sum_n W^{a\,(n)}_\mu(x)\frac{f_n(y)}{\sqrt{\pi r_c}}. \tag{B.1}$$

Using this expansion and inserting $\langle H\rangle$, we find

$$\mathcal{L}_{\rm IR}\supset g_5 g_{5i}\frac{v^4}{16}\sum_{n,m}W^{a(n)}_\mu X^{i\,(m)\,\mu}\frac{f^a_n(z_v)}{\sqrt{\pi r_c}}\frac{f^i_m(z_v)}{\sqrt{\pi r_c}}. \tag{B.2}$$

If $a=3$, $X^{i\,(m)\,\mu}$ is $W^{3\,(m)\,\mu}$, $Z'^{(m)\,\mu}$, or $B^{(m)\,\mu}$. If $a=1$ or 2, $X^{i\,(m)\,\mu}$ is $W^{1,2\,(m)\,\mu}$ or $\widetilde{W}^{1,2\,(m)\,\mu}$. $g_{5i}$ is the appropriate coupling in each case, and $f^i_n(z_v)$ is evaluated at the IR brane.

With a left handed zero mode on each external line, $f^a_0(z_v)=1$ since zero-mode has a flat profile. Using (B.2) and Feynman gauge, diagrams like Fig. 1 are

$$g_5^2 g_{5i}^2\frac{v^4}{16}\sum_n\frac{i}{(\pi r_c)^2}\frac{f^i_n(z_v)f^i_n(z_v)}{q^2-m_n^2}\eta^{\mu\nu}. \tag{B.3}$$

Here, $m_n$ is the mass of the $n^{\rm th}$ KK mode, and $q^2$ is the four momentum in the KK propagator.

Summing over eigenfunctions, Fig. 1 is then

$$g^2 g_{5i}^2\frac{v^4}{16}\sum_n\frac{i}{\pi r_c}\frac{f^i_n(z_v)f^i_n(z_v)}{q^2-m_n^2}=i\frac{v^4}{16}g^2 g_{5i}^2 G^{5D}_{q\,i}(z_v,z_v). \tag{B.4}$$

$G^{5D}_{q\,i}(z_v, z_v)$ is given in equations (A.1) and (A.2)) and $g = g_5/\sqrt{\pi r_c}$ is the $4D$/zero-mode gauge coupling. Choosing to write the graphs in terms of $G^{5D}_{q\,i}(z_v, z_v)$ automatically does the sum over KK modes and considers the different boundary conditions on the $f^i_n(z_v)$ for each field. With this prescription, the Feynman rules for Fig. 1 are simply $g g_{5i} \frac{v^2}{4}$ at each vertex, and $i G^{5D}_{q\,i}(z_v, z_v)$ for the gauge boson propagator.

The propagator for the zero-mode is:

$$G^{(0)}_{q(++)} = \frac{1}{q^2 \pi r_c} \tag{B.5}$$

There is no zero-mode for $(-,+)$ boundary condition.

For the tree-level contribution to $T$ from gauge-Higgs sector, we need propagator at zero momentum with *zero*-mode *subtracted*. For $(+,+)$ boundary conditions, this is (up to $O\left(z_h^2/z_v^2\right)$)

$$G^{5D}_{q=0(+,+)}(z_v, z_v) - G^{(0)}_{q(++)} \;=\; \frac{1}{\pi r_c}\left(\frac{z_v^2}{2} - \frac{z_v^2 k \pi r_c}{2} - \frac{z_v^2}{4 k \pi r_c}\right) \tag{B.6}$$

and for $(-,+)$ boundary condition with a bulk mass $M$ (up to $O\left(z_h^2/z_v^2,\ M^4/k^4\right)$):

$$G^{5D}_{q=0(-,+)}(z_v, z_v) \;=\; -\frac{z_v^2}{2 z_h}\left(1 - \frac{M^2}{4k^2}\right) \tag{B.7}$$

# C Fermionic operators

As in the calculation of the diagram in Fig. 1, we convert the sum over KK modes in Figs. 3 and 2 into a $5D$ propagator. For compositeness Feynman diagram (Fig. 3), we need propagator at zero momentum with *zero*-mode *subtracted*. For $(+,+)$ boundary conditions this is

$$G^{5D}_{q=0(++)}(u, v) - G^{(0)}_{q(++)} \;\approx\; \frac{1}{\pi r_c}\left(\frac{u^2}{4} - \frac{u^2 \log \frac{u}{z_h}}{2} + \frac{v^2}{4} - \frac{v^2 \log \frac{v}{z_v}}{2} - \frac{z_v^2}{4 k \pi r_c}\right) \tag{C.1}$$

Similarly, for fermion-Higgs operator Feynman diagram (Fig. 2), we need propagator from TeV brane to $z$ for zero momentum (again, with zero-mode subtracted):

$$G^{5D}_{q=0(++)}(z, z_v) - G^{(0)}_{q(++)} \;\approx\; \frac{1}{\pi r_c}\left(\frac{z^2}{4} - \frac{z^2 \log kz}{2} + \frac{z_v^2}{4} - \frac{z_v^2}{4 k \pi r_c}\right) \tag{C.2}$$

for $(+,+)$ boundary condition and

$$G^{5D}_{q=0}(z, z_v) \;\approx\; \frac{z_h}{2}\left(1 - \frac{z^2}{z_h^2}\right) \tag{C.3}$$

for $(-,+)$ boundary condition with $M=0$.

The $c$-dependent wavefunction of fermion zero-mode is (see, for example, [12, 34]):

$$\begin{aligned}\Psi(x,z) &\ni \psi^{(0)}(x)\chi_0(c,z), \text{ where}\\ \chi_0(c,z) &= \sqrt{\frac{1-2c}{z_h\left(e^{k\pi r_c(1-2c)}-1\right)}}\left(\frac{z}{z_h}\right)^{2-c}\end{aligned} \tag{C.4}$$

# D Gauge KK masses and couplings

The masses of gauge KK modes with $(+,+)$ boundary condition are given by

$$\frac{J_0\left(m^{(n)}_{gauge}z_h\right)}{Y_0\left(m^{(n)}_{gauge}z_h\right)} = \frac{J_0\left(m^{(n)}_{gauge}z_v\right)}{Y_0\left(m^{(n)}_{gauge}z_v\right)} \tag{D.1}$$

so that, for $m^{(n)}_{gauge}z_h \ll 1$, we get $m^{(n)}_{gauge}z_v \approx$ zeroes of $J_0 + O\left(1/\left[\log m^{(n)}_{gauge}z_h\right]\right)$. In particular, the mass of the lightest gauge KK mode is given by

$$m^{(1)}_{gauge} \approx 2.45 z_v \tag{D.2}$$

The masses of gauge KK modes with $(-,+)$ boundary condition and a bulk mass $M$ are given by

$$\frac{J_\nu\left(m^{(n)}_{gauge}z_h\right)}{Y_\nu\left(m^{(n)}_{gauge}z_h\right)} = \frac{\tilde{J}_\nu\left(m^{(n)}_{gauge}z_v\right)}{\tilde{Y}_\nu\left(m^{(n)}_{gauge}z_v\right)} \tag{D.3}$$

so that, for $m^{(n)}_{gauge}z_h \ll 1$, we get $m^{(n)}_{gauge}z_v \approx$ zeroes of $J_0 + M^2/\left(4k^2\right) + O\left(\left[z_h m^{(n)}_{gauge}\right]^2,\ M^4/k^4\right)$.

We compute couplings of gauge KK modes to Higgs/TeV-brane localized fields by writing the TeV-brane-to-TeV-brane propagator with zero-mode subtracted (at zero-momentum) as a sum over KK modes multiplied by respective couplings (as in Eq. (B.4)):

$$\sum_n \frac{\left(f^i_n\right)^2(z_v)}{\pi r_c}\frac{1}{m^{(n)\,2}_{gauge}} \approx \frac{z_v^2}{2z_h} \tag{D.4}$$

using Eq. (B.6). Using the above spectrum for KK modes gives $f^i_n(z_v) \approx \sqrt{2k\pi r_c} f_0(z_v)$ (for both $(+,+)$ and $(-,+)$ boundary conditions), i.e., coupling of gauge KK modes to TeV-brane localized fields is enhanced compared to that of zero-mode by $\sqrt{2k\pi r_c}$.

# E Spectrum and couplings to Higgs (at TeV brane) of $t_R$ and $(t,b)_L$ $(+,+)$

There is a chiral zero-mode and the masses of KK modes, $m_n$ are given by (see, for example, [19]):

$$\frac{J_{\alpha\mp 1}\left(m_n z_h\right)}{Y_{\alpha\mp 1}\left(m_n z_h\right)} \;=\; \frac{J_{\alpha\mp 1}\left(m_n z_v\right)}{Y_{\alpha\mp 1}\left(m_n z_v\right)} \equiv -b_\alpha(m_n), \tag{E.1}$$

where the upper (lower) signs are for $c > -1/2$ ($c < -1/2$) and $\alpha = |c+1/2|$. We will need masses of lightest KK modes only so that henceforth we assume $m_n z_h \ll 1$.

For $-1/2 < c < 1/2$, we get $-1 < (\alpha - 1) < 0$ so that, using $Y_\nu = 1/\sin\nu\pi \;\left(J_\nu \cos\nu\pi - J_{-\nu}\right)$, we see that $Y_{\alpha-1}\left(m_n z_h\right) \propto \left(m_n z_h\right)^{\alpha-1}$ just like $J_{\alpha-1}\left(m_n z_h\right)$. Thus, LHS of Eq. (E.1) does not have a "convenient" limit. So, we use the above relation for $Y_\nu$ to show that $Y_{\alpha-1}$ can be replaced by $J_{-\alpha+1}$ (on both sides) in the above equation[7]. Then, for $-1/2 < c < 1/2 - \epsilon$ (for a suitable $\epsilon$: see below), LHS of Eq. (E.1) $\sim \left(m_n z_h\right)^{2(c-1/2)} > 1/\left(m_n z_h\right)^{2\epsilon} \gg 1$ so that $m_n z_v \approx$ zeroes of $J_{1-\alpha} = J_{-c+1/2}$. Therefore, the lightest KK mass $\sim z_v^{-1}$ so that, in order to validate the assumption that LHS of Eq. (E.1) $\gg 1$, we need $\left(z_h/z_v\right)^\epsilon \ll 1$: we see that $\epsilon \gtrsim 0.1$ suffices. Whereas, for $c < -1/2 - \epsilon$, LHS of Eq. (E.1) $\sim \left(m_n z_h\right)^{2(\alpha+1)} \sim 0$ and so we get $m_n z_v \approx$ zeroes of $J_{\alpha+1} = J_{-c+1/2}$.

The wavefunctions of KK modes are given by (see, for example, [12, 34]):

$$\Psi(x,z) \;\ni\; \frac{\left(\frac{z}{z_h}\right)^2}{\sqrt{\pi r_c}} \sum_{n\neq 0} \psi^{(n)}(x)\chi_n(z), \tag{E.2}$$

$$\chi_n(z) \;=\; \frac{\sqrt{\frac{z}{z_h}}}{N_n}\left[J_\alpha\left(m_n z\right) + b_\alpha(m_n) Y_\alpha\left(m_n z\right)\right], \tag{E.3}$$

$$\frac{1}{\pi r_c}\int_{z_h}^{z_v} dz\, \chi_n^2(z) \;=\; 1, \tag{E.4}$$

and

$$N_n^2 \;=\; \frac{1}{2\pi r_c z_h}\left[z_v^2\left[J_\alpha\left(m_n z_v\right) + b_\alpha(m_n) Y_\alpha\left(m_n z_v\right)\right]^2 - z_h^2\left[J_\alpha\left(m_n z_h\right) + b_\alpha(m_n) Y_\alpha\left(m_n z_h\right)\right]^2\right]. \tag{E.5}$$

[7] Or we can use $J_{-\alpha}$ instead of $Y_\alpha$ as the Bessel function of the second kind in the wavefunctions of KK modes to get the same result.

In particular, we need coupling of KK modes to Higgs to determine mass insertions in Fig. 6, i.e., wavefunction of KK modes *at TeV* brane:

$$\Psi(x, z_v) = \sum_n \frac{\psi^{(n)}(x)}{\sqrt{1-\delta_n}} \frac{\sqrt{2} z_v^{\frac{3}{2}}}{z_h^2}, \tag{E.6}$$

where

$$\delta_n = \frac{z_h^2 \big[ J_\alpha (m_n z_h) + b_\alpha(m_n) Y_\alpha (m_n z_h) \big]^2}{z_v^2 \big[ J_\alpha (m_n z_v) + b_\alpha(m_n) Y_\alpha (m_n z_v) \big]^2} \tag{E.7}$$

We get $\delta_n \approx 0$ for $m_n z_h \ll 1$.

It is useful to define $f(c)$ to be ratio of wavefunction *at TeV brane* of KK mode and zero-mode (using Eqns. (C.4) and (E.6)):

$$f(c) \approx \sqrt{\frac{2}{1-2c}} \tag{E.8}$$

for $c < 1/2 - \epsilon$ and using $\delta_n \approx 0$. Thus, $t_L^{(n\neq 0)} t_R^{(0)}$ "mass insertion" is $m_t f(c_L)$ and $t_L^{(n\neq 0)} t_R^{(m\neq 0)}$ mass insertion is $m_t f(c_L) f(c_R)$ – these mass insertions are used in the calculation in section 6.1.

# F Spectrum and couplings to Higgs (at TeV brane) of $b'_R$ $(-,+)$

For $(-,+)$ boundary condition, there is no zero-mode and the spectrum of KK modes is given by [19]:

$$\frac{J_\alpha (m_n z_h)}{Y_\alpha (m_n z_h)} = \frac{J_{\alpha \mp 1} (m_n z_v)}{Y_{\alpha \mp 1} (m_n z_v)} \equiv -b_\alpha(m_n). \tag{F.1}$$

An analysis similar to the one above shows that, for $c > -1/2 + \epsilon$, $m_n z_v \approx$ zeroes of $J_{c-1/2}$, whereas, for $c < -1/2 - \epsilon$, $m_n z_v \approx$ zeroes of $J_{-c+1/2}$.

In addition, for $c < -1/2 - \epsilon$, we can show that there is a mode much lighter than $1/z_v$ (when arguments of both LHS and RHS of Eq. (F.1) are small) given by $m_n z_v \approx 2\sqrt{\alpha(\alpha+1)} \, (z_h/z_v)^\alpha \ll z_v^{-1}$.

The expressions for wavefunctions of KK modes are similar to those for $(+,+)$ KK modes, except:

$$N_n^2 = \frac{1}{2\pi r_c z_h} \Big[ z_v^2 \big[ J_\alpha (m_n z_v) + b_\alpha(m_n) Y_\alpha (m_n z_v) \big]^2 - z_h^2 \big[ J_{\alpha \mp 1} (m_n z_h) + b_\alpha(m_n) Y_{\alpha \mp 1} (m_n z_h) \big]^2 \Big] \tag{F.2}$$

so that

$$\delta'_n = \frac{z_h^2 \left[ J_{\alpha \mp 1}\left(m_n z_h\right) + b_\alpha(m_n) Y_{\alpha \mp 1}\left(m_n z_h\right) \right]^2}{z_v^2 \left[ J_\alpha\left(m_n z_v\right) + b_\alpha(m_n) Y_\alpha\left(m_n z_v\right) \right]^2}. \tag{F.3}$$

For $c > -1/2 + \epsilon$, we get $\delta'_n \approx 0$ so that wavefunction *at TeV* brane is *same* as for $(+,+)$ KK modes. Thus, $b_L^{(m \neq 0)} b_R'^{(n)}$ mass insertion is $m_t f(c_L) f(c_R)$ – these mass insertions are used in the calculation in section 6.1.

For $c < -1/2 - \epsilon$, for the "very light" mode, we get $\delta' \approx (c + 1/2)/(c - 1/2)$ so that (using Eqs. (C.4) and (E.6)) very light mode has the *same* wavefunction on TeV brane as *zero*-mode of $t_R$, whereas $\delta'_n \approx 0$ for the other modes of $b'_R$.

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